to the galaxy. So these get bent more, and the result is that the whole beam is given a little extra divergence.

The net effect is the same as with a glass lens: the extra divergence makes the image appear closer, and therefore brighter. The figure shows that the overall bending of the path of the light moves the image *away* from the galaxy that acts as the lens.

Why images get brighter

In this section: we explain why a diverging lens makes images brighter. This applies to glass lenses as well as gravitational lenses.

It is not difficult to see from Figure 23.3 on the previous page that the image should seem closer, but why should it be brighter? After all, the light rays are diverging faster when they reach the astronomer; should not that make the image dimmer?

This apparent paradox is present in the theory of the glass lens, too. Its resolution is to realize that the brightness of the image is not represented by the rays in Figure 23.3. They show the light from one point on the star as it reaches many places in the telescope. The brightness of the image, on the other hand, is determined by the rays that reach a given place on the telescope from different places on the star: how much light do they bring from the star?

As a first step in understanding what happens, we discuss what happens when the observer looks through the telescope at a distant galaxy, rather than a tiny star. Consider a small pencil of rays from the observer that reach the galaxy, as in Figure 23.4. Suppose in fact that the pencil is so narrow that when it reaches the galaxy it covers only a small part of the surface of the galaxy. (These are the rays that, say, will bring the light to one **pixel** of the image of the galaxy on the observer's photographic plate.) Since the lens has made the pencil diverge more than it would have if the lens were not there, these rays intersect more of the surface of the galaxy than if the lens were not there. This tends to bring *more* light into the observer's eye. In fact, it exactly compensates the divergence we noted in the first paragraph of this section: the light from the surface of the galaxy is indeed being spread out more by the lens, so less of it reaches us from any part of the galaxy. But the lens allows us to fit more of the surface of the star into our pencil of rays, with the following net result.

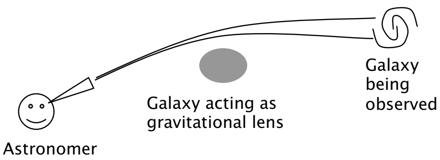

Figure 23.4. The brightness of the image depends on how much light from the source arrives at a point in the astronomer's telescope. Therefore we draw light rays that originate at the astronomer, the opposite of the rays we drew for working out where the lensed image was, in Figure 23.3 on the previous page. The divergence of the rays means that they cover more of the surface of the object (a galaxy in this diagram) than they would have covered if the lens had not been there. This compensates the divergence of the rays from the galaxy, so that the brightness of any small angular part it is the same as if the lens were absent.

> A pencil of rays with a given angular width receives the *same* amount of light from the galaxy regardless of whether the lens is present or absent, provided that the pencil is smaller than the angular size of the galaxy.

Naturally, this is true only if the lens is transparent; we don't worry here about absorption or scattering of the light by the lensing objects.

Therefore in Figure 23.4 we draw the same situation, but we trace rays back from the astronomer to the star. They pass the galaxy and are lensed in exactly the same way, which means they are given a little extra divergence. The effect of this is that when they reach the star, they occupy *more area* on the star than they would have if the galaxy had not been there.

> The extra brightness of the image of the star comes from the fact that more of the star is contributing light to this point at the entrance to the telescope: the image of the star is brighter because more light from it arrives at the telescope than if the lens were not there.

The word that astronomers use for the amount of light received from a piece of the surface of an object into a given angle at the telescope (into a given pixel) is **surface brightness**. We have shown that the surface brightness of an object is unchanged by the lens. This applies to lenses in ordinary optics as well as to gravitational lensing.

But still, why do stars look brighter though a gravitational lens? Have we not just proved that they will be the same brightness? No, we have proved that a piece of the star, covering a given angular size in our observation, will be just as bright as before. But when we consider the whole star, we need to take account of the fact that the size of the image of the star is larger than without the lens, because the diverging lens has made the star appear closer. The star occupies a larger angular size on the sky. Since it has the same surface brightness, we get more light in total from it.

This effect is particularly important for stars, whose angular sizes are so small that astronomers cannot resolve individual parts of their surfaces. In a photograph, it is not possible to tell that the star is in fact larger, since its size is still too small for the telescope to resolve. All we see in the photograph is that there is more light from the star: it is brighter. Galaxies that are big enough to resolve in a photograph, on the other hand, are no brighter in a given area of the photograph than without the lens. They are simply bigger.

You might now ask, what happens to conservation of energy? If there is more light arriving at the telescope from the star, where is this energy coming from? Clearly, the star is not making any extra light, nor is the lens. The extra light in the telescope is light that would have gone elsewhere but is being re-directed by the galaxy into the telescope. Therefore, some other astronomer must be losing the light that should have arrived. Where is he?

The position of the astronomer in Figure 23.4 is not particularly special. Any astronomer will get a little extra light if the light passes the galaxy. The astronomers who lose out are on the other side of the star. The galaxy's gravitational attraction has pulled a little of the light that is meant to go to the right in the diagram and is sending it to the left, and this allows the re-distribution of light that makes the image brighter.

If the lens were more complicated, like the one we are about to look at in Figure 23.6 on the next page, then the situation is also more complicated. Different parts of an image can brighten up at the expense of neighboring parts, as well as at the expense of the unfortunate astronomers in the other half of the Universe.

Making multiple images: getting caustic about light

In this section: we learn that the number of images is related to the way the light rays from the object intersect one another in caustics.

If you bought a new camera and found, when you had developed the first roll of film, that there were two images in the same photo of your grandmother holding her pet dog, you would feel cheated and you would demand your money back. But when gravitational lenses do this, we all get excited! In fact, we will see that gravitational lenses typically give you *three* images of your grandmother, and in one of them she is left-handed!

▷Of course, if your grandmother really is left-handed, then in the third image she will be right-handed!

In this section we explore one of the extra images, called the second direct image. Its existence is easy to understand and of most interest for applications of lensing. In the next section we will investigate the Einstein ring, an important radius around the lens, which is the key to understand microlensing and the discovery of MACHOS (see page 172). After that we will thread our way through the subtle reasoning that shows that if there is a second image then there is also a third, in which left and right are reversed.

What normally happens is that there is only one image, a little brighter. Other rays, that pass on the other side of the lensing galaxy, do not deflect enough to reach the astronomer. This is illustrated in Figure 23.5 on the following page. Notice,