> such phenomena, scientists hope to learn what some of the basic features of quantum gravity may be.

Hawking radiation: black holes are truly black bodies

In this section: remarkably, like other perfect absorbers, black holes actually radiate a black-body spectrum. This is a purely quantum effect. The radiation from astrophysical black holes is undetectably small, but in the early Universe small black holes might have formed that would explode today.

Negative-energy fluctuations may be speculative when it comes to making wormholes, but they are well-established in another aspect of gravity: they turn black holes into black bodies.

Recall our discussion of black bodies in Chapter 10. There we saw that any body that absorbs all the light that falls on it is a black body, and when heated to a given temperature T it will give off a characteristic spectrum of radiation. Now, a black hole certainly absorbs all the light that falls on it, so it is a black body. But we have seen that nothing from inside can get out of a black hole, so it would appear that it cannot be a source of radiation. Black holes therefore don't seem to fit comfortably into thermal physics.

Figure 21.9. *Stephen Hawking is one of the most well-known physicists of our time. He has been able to reach the general public with deep questions that are at the current limits of physical theory. He has strongly influenced the development of theoretical physics, deepening the understanding of black holes and making a major step toward quantum gravity with his discovery black holes must emit thermal radiation. Photo courtesy S W Hawking.*

However, black-body radiation is a quantum phenomenon: Planck invented his constant in order to describe it. Fittingly, therefore, when the British physicist Stephen Hawking (b. 1942) studied the quantum theory of electromagnetism near black holes, he found that black holes actually emit radiation, that in fact has a black-body spectrum.

How can black holes emit radiation? It should be no surprize that the answer lies in quantum uncertainty. All over spacetime the quantum electromagnetic field is undergoing the little negative-energy quantum fluctuations that we considered above. Normally they are harmless and invisible, because the negative-energy photons disappear as quickly as they form. But near the horizon of a black hole, it is possible for such a photon to form outside the hole and cross into it.

Once inside, it is actually viable: as we remarked earlier, it is possible to find trajectories for photons inside the horizon that have negative total energy. So such a photon can just stay inside, and that leaves its positive-energy partner outside on its own. It has no choice but to continue moving outwards. It becomes one of the photons of the **Hawking radiation**.

> In this picture, nothing actually crosses the horizon from inside to out. Instead, the negative-energy photon falls in, freeing the positive-energy photon. The net result of this is that the hole loses mass: the negative-energy photon makes a negative contribution to the mass of the hole when it goes in.

Once we accept that black holes can radiate, then it is not hard to estimate the wavelength of the radiation that they emit. The only length-scale in the problem is the size of the horizon. A photon with a wavelength λ equal to the radius of the black hole has (ignoring the curvature of spacetime in this simple argument) an energy equal to

▷If you worry about our choice of wavelength here, consider that the uncertainty in the location of a photon is about one wavelength. Very short wavelengths are localized outside the hole and fluctuations can't cross in time. Very long wavelengths hardly notice the hole and have little chance of finding their way inside.

$$E = h\nu = h\frac{c}{\lambda} = hc\frac{c^2}{2GM} = \frac{hc^3}{2GM}.$$

If black holes are indeed black bodies, absorbing everything that falls on them and emitting light, then their temperature T should be at least approximately related to this energy by setting $E = kT$, leading to the following estimate of the temperature of a black hole,

$$T = \frac{hc^3}{2kGM}.$$

Now, our argument cannot be expected to be exact, since we had no reason to take the wavelength equal to the radius of the hole rather than, say, its diameter or circumference, and since we must expect that the details of quantum theory and spacetime curvature will not be encapsulated in such a simple dimensional argument.

Investigation 21.4. The decay of a black hole

Here we study how long it takes a black hole to lose a significant amount of mass because of Hawking radiation. The temperature of a Schwarzschild black hole given in Equation 21.13 allows us to calculate the luminosity from the standard formula for a black body, Equation 10.3 on page 116,

$$L = \sigma A T^4,$$

where σ is the Stefan–Boltzmann constant, defined in Equation 10.4 on page 116, and A is the area of the surface that radiates. The surface in this case is the horizon, so the area is the area of a sphere with radius $2R_g$. (Recall that the radial coordinate that Schwarzschild used is the one that measures the circumference of the sphere, not the distance to its center. Therefore, we can be confident that the area of the sphere that is the horizon is given by the usual formula for spheres, even if we don't know what space is like inside the horizon.) This gives

$$A = 4\pi(2R_g)^2 = 4\pi(2GM/c^2)^2 = (16\pi G^2/c^4)M^2.$$

Combining this with all the other quantities gives the luminosity of the black hole, and grouping terms in a special way, gives

$$L_{bh} = \frac{1}{30720\pi^2}\,\frac{ch/G}{M^2}\,\frac{c^5}{G}.$$

It is instructive to take this expression apart. The first factor is, of course, just a pure number. The second contains, in its numerator, the quantity ch/G. It is the square of the Planck mass $m_{\rm Pl}$, defined in Equation 12.20 on page 146, which we have discussed elsewhere in this chapter. So the second factor is dimensionless, being the ratio of the squares of two masses. The third factor is the Einstein luminosity, also discussed in the body of this chapter.

The Einstein luminosity is large, but the black hole only approaches this luminosity when its mass is as small as the Planck mass. For an ordinary hole, the factor in $1/M^2$ reduces the luminosity drastically. For example, a $10M_\odot$ black hole radiates 10^{-30} W!

The lifetime of a black hole can be estimated to be $Mc^2/L_{\rm bh}$; this is an overestimate, since it assumes the luminosity will be constant in time, whereas it increases. But the increase is gradual, and so the estimate will be accurate to a factor of something like two. (A detailed calculation shows that the true lifetime is one-third of this estimate, which is not much error when we are dealing with such huge times.) For the $10M_\odot$ black hole, this estimate gives 2×10^{78} s, an unimaginably long time!

What is the mass of the hole that will just decay in the age of the Universe, about 10^{10} y, so that if these were formed in the early Universe, we would be seeing their explosions now? Just set the lifetime, $Mc^2/L_{\rm bh}$, to this value and solve for M. The answer is that the hole should have a mass of about 10^{12} kg. This hole is too small to form today or at any time since galaxies formed, but perhaps in the very early universe conditions were different. There is no observational evidence for such holes, however.

Exercise 21.4.1: *Hawking radiation*

Perform the computations indicated in this investigation. Then find out how much time the hole has remaining when its temperature is high enough to produce electrons in its radiation (this will require kT to exceed $m_e c^2$).

Nevertheless, our answer is only a factor of $8\pi^2$ larger than the one that Hawking found, which is now called the **Hawking temperature** $T_{\rm H}$:

$$T_{\rm H} = \frac{hc^3}{16\pi^2 kGM} = 6 \times 10^{-8}\left(\frac{M}{M_\odot}\right)^{-1}\ {\rm K}. \qquad (21.13)$$

This is so small for stellar-mass and supermassive black holes that it has little relevance to astrophysics. But Hawking's discovery is widely regarded as one of the first real steps toward a quantum theory of gravity. Although we have no such theory, many physicists expect that it must predict the Hawking radiation.

Through this radiation, black holes gradually lose mass. The smaller they get, the higher their temperature goes (by Equation 21.13), so the loss of mass accelerates. In Investigation 21.4 we use our knowledge about black-body radiation to calculate the lifetime of a black hole. For a one-solar-mass black hole, it is about 10^{67} y!

But smaller holes have shorter lifetimes. The mass of a hole that has a lifetime equal to the age of the Universe, about 10^{10} y, is 10^{12} kg. (See Investigation 21.4.) We have seen earlier that holes of this mass cannot form today, but it is conceivable that such **primordial black holes** did form by random fluctuations in the very early Universe.

These primordial black holes would be ending their lives today in an explosion. The amount of energy released in the last second of a black hole's life equals the energy equivalent of the mass of a black hole whose lifetime equals one second. This is a hole of mass about 10^6 kg, which converted into energy gives about 10^{23} J.

▷The release of this much energy in one second might be observable: it is only a fraction of a percent of the solar luminosity, but it would come out in gamma-rays; this does not explain the observed gamma-ray bursts. They have a luminosity that is up to 10^{22} times larger than this!

The Hawking radiation has linked black hole physics to two other, very different branches of physics: thermal physics and quantum gravity. When an unexpected result makes such links, they must be fundamental. In the next sections we will look at them further and learn why physicists find the Hawking radiation such a deeply satisfying result.

Black hole entropy: a link to nineteenth century physics

In this section: Hawking radiation allows physicists to define the entropy of a black hole. Entropy was introduced into thermal physics in the nineteenth century, and measures the disappearance of information. Since black holes swallow almost all the information that falls into them, they have extremely large entropy. Hawking radiation allows them to exchange entropy with other systems.

In this book we have discussed many aspects of gas dynamics in astronomy, but we have not yet studied the fundamental concept of entropy. Our study of black holes has led us to the point were it is now time to fill in this gap. The entropy of black holes is a remarkable illustration of the unity of the fundamental concepts of physics across different disciplines.

Entropy fundamentally has to do with measuring how much *information* a system contains. Information is related to order. An ordinary gas is highly disordered, its atoms moving in a random manner that is well described by only a few numbers, such as the density, composition, and temperature of the gas. A crystal lattice, on the other hand, has more structure, and correspondingly requires more information to describe it: the spatial arrangement of the atoms, their separations, the locations of any impurities, and so on. If a system is ordered, then it requires more information to describe it than if it is disordered. Entropy measures disorder. A highly ordered system has *low* entropy, and a messy system has *high* entropy.

Entropy was first introduced into gas dynamics by the German physicist Rudolf Clausius (1822–1888), but he did not associate it with disorder. This fundamental step was the greatest triumph of Boltzmann, whom we met in Chapter 7. He was able to show that his statistical mechanics, from which he could derive the pressure–density relation for gases, could also give a deeply satisfying definition of Clausius' entropy. Basically, Boltzmann showed that one could compute the entropy by counting the number of different ways that the molecules of a gas could be arranged to produce the same overall state of the gas: the same pressure, temperature, and density. This number is huge, of course, and the entropy is essentially the logarithm of it times the Boltzmann constant k.

Clausius had introduced entropy in order to describe heat flow. We have not needed to discuss it before because most of the fluid dynamics we have discussed in this book has been without heat conduction. In astronomy, heat flow is usually a secondary effect. But in engines and other technological systems, heat conduction is central to the function. Clausius originally defined the change in entropy of a system as the heat energy it absorbed divided by the temperature at which it absorbed the heat. When a system does things without losing heat, such as a gas expanding a piston in an idealized non-conducting environment, then the entropy of the gas did not change.

Since systems can gain or lose heat, their entropy can increase or decrease.

> The remarkable discovery of Clausius was that – essentially because heat always moved from high-temperature regions to low-temperature ones – the total change in entropy, summed over all the parts of a system that were exchanging heat with one another, was always *positive*. The entropy of the universe is always increasing.

This could be shown mathematically, but early physicists had no fundamental explanation for it.

Boltzmann showed that this was to do with disorder. Individual systems can get more ordered – I can clean up my desk once in a while, maybe – but the universe as a whole gets more disordered. When I clean my desk I expend so much energy that the entropy of the air in the room and of the chemicals in my body dramatically increase. (That's why I do it so rarely!)

> It is one of the deep mysteries of the world that entropy increases, *disorder* increases, as time goes on. This so-called thermodynamic **arrow**