

Here is the argument in detail. Suppose we have a gas of neutrons whose density is much less than nuclear density. Then when neutrons collide with one another, as must happen at random all the time, they will have a tendency to stick together and form large nuclei; the extra pressure of neutrons from outside will make these condensations bigger than ordinary nuclei like uranium. So the gas will be a mixture of free neutrons and big nuclear lumps. Now consider what happens when the gas is compressed. Collisions become more likely, and the result will be that many neutrons will get stuck in lumps and not contribute to the gas pressure: remember from Chapter 7 that the pressure of a gas depends on its temperature and the number of particles in the gas, not on the masses of the particles. If free neutrons are lost to the lumps, then the pressure will not build up fast when the gas is compressed, and it will not be able to hold itself up against gravity. The collapsing core of a giant star in a supernova explosion therefore continues to collapse well after neutrons have been formed.

When the density of the collapsing core reaches nuclear density, the lumps all merge into a smooth “pudding”, and further compression sees a rapid increase in pressure from the hard-core repulsion. Collapse stops, and a neutron star with the density of an ordinary nucleus is formed.

Now we see that the incredibly small size of neutron stars compared to the Sun is not so hard to explain. In normal matter, nuclei are separated by the huge distances occupied by the intervening electrons. In a neutron star, Nature has simply managed to remove all that wasted space, and put all the nuclei right up against one another.

It takes a whole star to do the work of 100 neutrons

This argument tells us what the density of a neutron star should be, but it does not tell us about the mass. It does not tell us whether this phenomenon should occur with stars or with basketballs. Why are there neutron stars, and not neutron basketballs? Or neutron galaxies?

The answer, of course, is gravity. To see why, let us imagine trying to make a neutron basketball. If one takes a heavy nucleus, say of uranium, and tries to build it up into a neutron basketball by adding one nucleon at a time, something goes wrong: as soon as a nucleon is added, the nucleus spits it out again, or worse still the whole nucleus divides in half. This is *radioactivity*.

Heavy nuclei decay through radioactivity because they are unstable. This happens basically because of the second feature of the nuclear force that did not come into our previous discussion but which is obvious if we look at everyday life from the right point of view: even the attractive part of the nuclear force is of very *short range*. We can see that it must be short range, since essentially all the properties of ordinary materials can be explained by using just the electric and magnetic forces that electrons and nuclei exert on one another through their electric charges. The nuclear forces are intrinsically strong, since they can hold all the protons in a nucleus together, despite their mutual electric repulsion. But they do not extend very far from the nucleus, since they do not influence chemistry. Unlike gravity and the electrostatic force, which fall off as $1/r^2$ as one goes away from the source, the nuclear force must fall off much more rapidly as one leaves the nucleus.

This means that, as one adds nucleons to a nucleus, there will come a point where nucleons on one side of a nucleus no longer feel the attraction exerted by those on the other side. Protons still feel the electrostatic repulsion of other protons, however, so if one adds protons to a sufficiently large nucleus, they will simply be pushed out

▷Readers who have read Investigation 8.8 on page 101 will have already gone through this argument about pressure in detail.

In this section: the neutrons in a neutron star are held together by gravity. We show here that gravity is only strong enough to replace the binding forces that hold nuclei together when there are as many nucleons as in a typical star. This coincidence is one of the deep mysteries of nature, because without neutron stars there would be no life on Earth.

again: the new proton feels a nuclear attraction from only a few nucleons, but a repulsion from all the existing protons.

If one adds neutrons, one avoids this repulsion, but one still runs into a problem: the Pauli exclusion principle (Chapter 12). As one adds more and more nucleons of either kind, the new ones cannot have the same low kinetic energies of the existing ones, since the existing ones have filled up all the low-energy quantum states. So new nucleons must have higher energies, and at some point these energies will be enough to escape from the attraction exerted by the nearby nucleons. At this point, the nucleus will accept no new nucleons. This happens at roughly an atomic number of about 210: nuclei with more than 210 neutrons and protons in total tend to be unstable. This is about the location of lead in the periodic table.

▷Lead is an abundant mineral on Earth because it has been produced by the radioactive decay of heavier nuclei over the ages.

So we are frustrated in our attempts to build a nucleus with the mass of a basketball by the short range of the nuclear forces. To hold a bigger nucleus together requires a long-range force, and the only candidate is gravity. Electric forces won't do, since like charges repel, and an equal mixture of positive and negative charges will exert no net long-range force. So only gravity can stabilize nuclei bigger than lead.

Yet gravity is a weak force, and the attraction it exerts within an ordinary nucleus is tiny compared to the other forces. Gravity can only provide the glue to hold together a large nucleus if the self-gravitational force of the nucleus is comparable to the nuclear forces between nuclei. This is going to require a large amount of mass.

▷Recall that the symbol "MeV" represents a million electron volts, which is $10^6 \text{ eV} = 1.6 \times 10^{-13} \text{ J}$.

We can in fact compute just how much mass is required by a relatively simple argument. It is observed experimentally that the "escape energy" of a nucleon from a nucleus is about 8 MeV, the same for most nuclei. This is the energy that has to be supplied to a nucleon to get it away from the nucleus, and nuclei become unstable when the exclusion principle forces new nucleons to have this energy inside the nucleus. A nucleon that has a kinetic energy of 8 MeV inside a nucleus has just enough speed to escape. We show in Investigation 20.1 that this escape speed is about 13% of the speed of light.

Now, gravity can prevent this escape if it raises the escape speed: if the escape speed from a large clump of neutrons exceeds this value, then the clump will be one big stable self-gravitating nucleus: it will be a neutron star. In Investigation 20.1 we show that a star with the density of a nucleus has an escape speed exceeding the nuclear escape speed if the mass of the star exceeds roughly $0.02M_{\odot}$.

An object with more than 2% of the mass of the Sun and the density of a nucleus has strong enough gravity to keep the nucleons bound together. This is our estimate of the minimum mass of a neutron star.

So there are no neutron basketballs.

Despite the simplicity of our argument, our estimated minimum mass is very close to the value of $0.1M_{\odot}$ that full calculations give in general relativity, using more sophisticated nuclear physics. Considering that we have bridged a gap of a factor of 10^{53} from a nucleus of mass, say, 10^{-25} kg to the mass of a star, to have come within a factor of five of the right result is close indeed!

▷Notice that our way of calculating the mass of the neutron star from the binding energy of a nucleus is essentially the same argument as we used in Investigation 8.3 on page 91 to calculate the minimum mass of an object in the Solar System that is round, from the binding energy of silicon dioxide.

What about the *maximum* mass of a neutron star? As for white dwarfs, the maximum mass for neutron stars is set by the balance between the inward pull of gravity and the amount of pressure the nuclear matter can sustain. In Investigation 12.6 on page 148 we calculated the maximum mass of neutron stars in the same way as we calculated the Chandrasekhar mass for white dwarfs, and we obtained the result that the maximum mass should be about five or six solar masses.

Investigation 20.1. Minimum mass of a neutron star: no basketballs

Experiments show that the energy required to remove a nucleon from an ordinary stable nucleus is about 8 MeV. From this energy it is possible to deduce an “escape speed” for a nucleon from the formula

$$K = \frac{1}{2}mv^2,$$

where K is the escape energy. Some arithmetic gives, using the mass of the proton for m (see Appendix A),

$$v_{\text{escape}} = 4 \times 10^7 \text{ m s}^{-1},$$

or 13% of the speed of light.

Now, we want the escape speed from the neutron star to exceed this. This speed is (in Newtonian gravity) $(2GM/R)^{1/2}$, where M is the mass of the star and R its radius. The star must have nuclear density ρ_{nuc} , which means that we can deduce its radius from its mass. Writing down the equation for the average density,

$$\rho = \frac{M}{\frac{4}{3}\pi R^3},$$

and solving for R , we find

$$R = \left(\frac{M}{\frac{4}{3}\pi\rho} \right)^{1/3}.$$

This gives a gravitational escape speed for a “nucleus” of mass M :

$$v_{\text{escape}} = 1.8G^{1/2}M^{1/3}\rho_{\text{nuc}}^{1/6}. \tag{20.1}$$

Using the value of the nuclear escape speed for v_{escape} here gives a minimum value for M , which is

$$M_{\text{min}} = 4 \times 10^{28} \text{ kg} = 0.02M_{\odot}. \tag{20.2}$$

Exercise 20.1.1: How big is the nuclear hard core?

Use the mass 1.67×10^{-27} kg of a nucleon and the density 2×10^{17} kg m⁻³ to calculate the volume occupied by each nucleon in a nucleus. If the nuclei are contained in cubical boxes, how big is each box? What is the size of the hard core, the irreducible radius of a nucleon?

Exercise 20.1.2: Calculating the minimum neutron star mass

Solve Equation 20.1 for M and use the value of ρ_{nuc} in the previous exercise to verify the minimum mass in Equation 20.2.

Exercise 20.1.3: What does a neutron star look like?

Taking the mass of a neutron star to be $1M_{\odot}$, what is its radius? What is the escape speed of a projectile leaving its surface? What is the speed with which a projectile falling from rest far away reaches the surface? What fraction of the rest-mass of such a projectile is its kinetic energy when it arrives at the surface? What is the orbital speed of a particle in a circular orbit just above the surface of the star? What is its orbital period? Do all calculations using Newtonian gravity, even though the speeds are relativistic.

Exercise 20.1.4: Thermal effects in neutron stars

If the binding energy of a nucleon is 8 MeV, what temperature would the star have to have in order to boil off a nucleon? Since the pressure support for the star comes from the hard-core repulsion and not from random thermal motions of the star, it is possible for stars to cool off after formation without changing their properties. Give an argument that a star is “cold” (thermal effects are unimportant for its structure) if its temperature is smaller than the one you have just calculated. Assume the star has a temperature of 10^6 K. What is its black-body luminosity? (See Equation 10.3 on page 116.) What is the wavelength at which it is brightest? (See Equation 10.9 on page 117.)

However, this is too simple an estimate, since Newtonian gravity is just not accurate enough for such compact stars. One needs to use general relativity to calculate their structure. We will do this in Investigation 20.3 on page 280, but for now we only point out that the effect of using relativity is to lower the maximum mass to somewhere between two and possibly three solar masses. Its actual value is not known: uncertainties in nuclear physics prevent reliable calculations.

We have learned that neutron stars can only exist in a rather restricted range of masses, between perhaps 0.1 and two solar masses. In fact, their lower limit in practice will normally be much larger, since a collapsing star will stop at the white dwarf stage if its mass is less than the Chandrasekhar mass. Neutron stars should only form if their masses are somewhat larger than $1M_{\odot}$. A collapsing star above the maximum mass will continue to collapse, and will form a black hole.

It is also interesting to ask what happens if we have a neutron star that subsequently gains or loses mass. If it gains mass, perhaps from a companion in a binary system, then it can be tipped over the maximum and it will collapse to a black hole. If it loses mass, again to a companion in a neutron star binary (see below), then when it reaches the minimum mass it will no longer be bound together and will undergo a catastrophic nuclear disintegration: it will *explode*.

The most remarkable and fortunate coincidence about these masses is that the maximum mass is larger than the Chandrasekhar mass. This coincidence allows

neutron stars to form in the first place. The maximum mass is a property of nuclear physics and general relativity. The Chandrasekhar mass depends on Newtonian gravity and atomic physics. We could imagine a Universe in which the nuclear repulsive core was smaller, so that neutron stars were denser and the effects of general relativity correspondingly greater, leading to a maximum mass smaller than the Chandrasekhar mass, which is unaffected by the nuclear hard core. In such a Universe, collapsing stars bigger than white dwarfs would form black holes directly. And in such a Universe, people would not exist.

The reason is that the nuclear hard-core repulsion plays a key role in the chain of events that leads to life on Earth. We have seen that the elements of which we are made were formed in stars, and that the heavier elements are spread into interstellar clouds by supernovae. Our Sun and Earth formed from clouds seeded with oxygen, silicon, and many other elements essential for life, by a long-ago supernova. But that supernova could not have happened if neutron stars could not form. If the collapsing core of the giant star that became the supernova could simply have continued to collapse to a black hole, then there would have been no “bounce”, no shock wave to blow off the envelope of the giant star. Instead, all the gas in the giant star would have fallen into the black hole. The vital elements carried by the supernova gases would never have left the star and found their way into our Solar System.

We owe our existence to the existence of neutron stars, and in particular to the neutron star that formed in that long-ago supernova. We must be thankful that Nature has arranged for the Chandrasekhar mass to be smaller than the maximum mass of neutron stars.

What would a neutron star look like?

Let us ask a few questions about the typical properties of a neutron star, just using the numbers we have obtained so far and assuming we can use Newtonian gravity.

Let us suppose the star has a mass of $1M_{\odot}$. This can't be far wrong, since a star with a mass less than the Chandrasekhar mass (see Chapter 12) will support itself at the density of a white dwarf and not collapse to a neutron star. So by taking a mass of $1M_{\odot}$ we are probably underestimating a little, but it will suffice to give us an idea of what the star will be like. In Exercise 20.1.3 on the preceding page you have the opportunity to do the calculations leading to the numbers below.

With a density of $2 \times 10^{17} \text{ kg m}^{-3}$, the star's radius will be about 13 km, or 8 miles: it would just cover Manhattan Island. The escape speed for a particle at its surface is $1.4 \times 10^8 \text{ m s}^{-1}$, or about half the speed of light. The orbital speed of a satellite at its surface is about 10^8 m s^{-1} , one-third of the speed of light. The period of such an orbit is 0.8 ms. This also sets the maximum spin rate of a neutron star: it could in principle rotate about 1000 times per second without flying apart.

The compactness of the star tells us also that the gravitational redshift of light from its surface will be significant. By the equivalence principle, an observer falling freely from far away will see no change in the frequency of light. Such an observer reaches the surface with the escape speed, $c/2$. The redshift seen by observers that remain at rest with respect to the star, then, is the same as that seen by an observer who is receding from a source of light at this speed. This will produce a lengthening of the wavelength of the light by at least a factor of two. If astronomers could see spectral lines in the radiation from a neutron star, they should be strongly redshifted.

These numbers can only be approximately correct of real neutron stars, since they tell us that typical speeds associated with the star are good fractions of the

In this section: from the properties of neutron stars we have already calculated, it is possible to make predictions about them. They are clearly dense and hot. They should emit X-rays and they should spin very fast. They could have strong magnetic fields.