***Investigation 8.2. How hot is the Sun?***

Here we want to infer the temperature of the Sun from the wavelength of the radiation it emits. We can rewrite Equation 8.5 on page 87 in terms of electron volts to give

$$E = 1.24\left(\frac{1\,\mu\text{m}}{\lambda}\right)\,\text{eV}. \tag{8.7}$$

As we mentioned in the text, there is a characteristic photon wavelength associated with any temperature. This is given by setting $E$ in Equation 8.7 equal to $\frac{3}{2}kT$ from Equation 7.3 on page 76, where $k$ is Boltzmann's constant. The result is that

$$\lambda \approx \left(\frac{9600\,\text{K}}{T}\right)\,\mu\text{m}. \tag{8.8}$$

In the center of the Sun, where the temperature reaches $10^7$ K, photons have typical equilibrium wavelengths of 0.001 μm and energies exceeding 1000 eV. Throughout the Sun, the energies of photons are enough to ionize hydrogen, and to strip electrons from other atoms too. Such a gas of electrons and ions is called a *plasma*.

**Exercise 8.2.1:** *Temperature of the Sun*

If one analyzes the colors of the Sun, one finds that the greatest amount of light is emitted in the *blue–green* region of the spectrum, around 0.5 μm. Show that this gives an *estimate* of the Sun's temperature of $T$ = 19 000 K. This is closer to the real temperature (5600 K) than our estimate in the text, but we will get a much better estimate by refining this technique in the next chapter. (The eye sees the Sun as yellow, not blue–green, partly because it has greater sensitivity to yellow light and partly because blue light is scattered by the atmosphere.)

Einstein was also the right man at the right time. Physicists were just learning how to probe the world of atoms and particles, and this world did not behave the way they expected, based on their experience with **macroscopic** objects. Einstein had an extraordinary ability to free his mind from prejudices and begin thinking in the new ways that atomic physics demanded. And not just to think, but to calculate, to make predictions that could be tested by experiment.

Interestingly, when Einstein received the 1921 Nobel Prize for physics, it was for the photoelectric effect. His work on relativity was explicitly excluded, since in the eyes of the awarding committee, it had not yet been sufficiently confirmed.

## *Gravity keeps the Sun round*

**In this section:** gravity singles out no special direction, nor does pressure, so stars and other large bodies are basically round. However, smaller bodies can be irregular in shape if chemical forces are significant. We calculate that any body with more mass than $1/1000^{th}$ of the mass of the Earth should be round: gravity should dominate chemistry. This fits well with observations in the Solar System.

We know the mass ($1.99 \times 10^{30}$ kg) and radius ($6.96 \times 10^{8}$ m) of the Sun, and from them we can work out that the mean density (mass divided by volume) of the Sun is about 1400 kg m$^{-3}$, or 1.4 times the density of water. To compress a gas whose interior temperature is several million degrees to beyond the density of water requires a great deal of force.

What is this force in the Sun? The answer can only be gravity, the gravitational attraction of one part of the Sun for another. This mutual attraction would, if unresisted, simply pull the material of the Sun inward towards a single point. The resistance to this collapse is provided mainly by gas pressure, and secondarily by the pressure provided by all the photons that are produced in the center of the Sun and gradually make their way outwards, scattering off electrons and nuclei in the Sun countless times as they go. The Sun exists in a state of balance between the outward push of gas and radiation pressure (the pressure of the photon gas) and the inward pull of gravity.

The photons produced in the center come from **nuclear reactions**, which are processes that change nuclei of some atoms into other nuclei. These reactions release a great deal of energy, and are the chief source of the energy that makes the Sun (and all other stars) shine. The energy from these reactions leaves the Sun in two main forms: as photons and as **neutrino**s, which are very light particles produced in many nuclear reactions. We will look at how nuclear reactions work in Chapter 11. For now, we just assume that there is an energy source in a small region around the center of the Sun.

The shape of the Sun is also determined by gravity. Gravity is, like pressure, an *isotropic* force, that is a force that has no preferred direction: the gravitational attraction exerted by any particle is the same in all directions. As long as the Sun is a balance between pressure and gravity, it can't help but form a ball that is round. If there were corners or other special places on its surface, then if we stood at the center of the Sun and looked outwards, there would be some directions different from other directions. Since there is nothing in gravity or pressure to single out these directions, they cannot exist: the Sun should be a sphere.

We have left out of this discussion three extra influences that *can* single out directions: rotation, a magnetic field, and the presence of a nearby gravitating body, such as a companion star in a binary system. We will discuss rotation below, since it does affect the Sun. Many stars have magnetic fields similar to that of the Earth, with a North and South magnetic pole. Pulsars, which we shall study in Chapter 20, have fields an incredible $10^{12}$ times stronger than the Earth's. If the field is strong enough, it can cause the star to have a distorted shape, particularly near the poles. The Sun's field is not that strong. Even the superprominence illustrated in the figure on page 85 contains a negligible amount of mass. Jupiter acts like a companion "star", distorting the shape of the Sun by tidal effects (see Chapter 5), but the effect is too small to measure. We will return to a more detailed discussion of the distorting effects of companions when we meet binary stars in Chapter 13. All in all, the Sun has no choice but to be spherical.

The arguments of the last paragraphs apply in fact to any astronomical bodies for which gas pressure and gravity are the main forces. But there are many astronomical bodies that are not round, because other effects dominate. Dust grains are whisker-shaped because of chemical forces; **asteroids** are irregular because the chemical forces that shape their rocks are as important as gravity; and on a very large scale, galaxies can be disk-shaped or cigar-shaped because the motions of large numbers of individual stars define their outlines.

In the case of asteroids, we can combine Boltzmann's understanding of the kinetic energy of a molecule (Chapter 7) with what we learned in Chapter 6 about escape speeds to answer the following elementary question.

> Why are planets and moons round, while ordinary rocks and even asteroids and the cores of comets have corners?

***Figure 8.2.*** *Phobos is one of the two moons of Mars, and has a mass smaller than the number we calculate in Investigation 8.3 to be the minimum mass for a rocky body to be forced by gravity to be round. Its irregular shape is consistent with our calculation. Image courtesy* NASA.

The answer has to do with melting. If, when a body was formed in empty space, temperatures got high enough to melt it, then gravity would, as we have just argued, make it spherical, as long as rotation, magnetic fields, and tidal effects were not too important. Since the atoms and molecules that form planets heat one another by colliding as they fall together, their kinetic energy when they collide must be comparable to their gravitational potential energy. Given a molecule of mass $m$, falling onto a planetary body of mass $M$ and radius $R$, its kinetic energy when it arrives will be something like $GMm/R$. The collisions randomize the direction of this energy, turning it into heat. The temperature, according to Boltzmann, will be given by setting $3kT/2$ equal to this energy. In Investigation 8.3 we put these expressions together and find that a rocky body in our Solar System should be round if its mass exceeds $3 \times 10^{21}$ kg. This number depends on some assumptions, especially that the body is composed of silicate

*Investigation 8.3. Why the Moon is round*

We do a little algebra here to find the minimum mass $M$ a body would need to have in order to melt as it forms. Once molten, gravity will shape it into a sphere. But if it does not melt, then it can have any irregular shape.

A molecule of mass $m$ falling onto the body will have a kinetic energy approximately equal to $GMm/R$, where $R$ is the size of the body (its radius, if it is spherical) . This is an approximation, and in fact the energy could be more, but we are only interested in a rough answer. The collision transforms this into random kinetic energy, or heat, with a temperature $T$ given by Boltzmann's relation

$$\frac{3}{2}kT \approx \frac{GMm}{R}.$$

We want to find the mass $M$ required to make $T$ high enough to melt the material. So we will assume that we know $T$, and set it equal to the melting point of rocks when we start doing numbers below. We shall also take $m$ to be the mass of a typical molecule in a rock crystal when we do the numerical calculation. So we would like to solve this equation for $M$ in terms of $T$ and $m$, but we don't yet know $R$, the size of the body. To get a sensible value for $R$ let us assume that we know the density $\rho$ of the body, which we will take later to be the density of rock. Knowing $\rho$ gives us a further approximation (again assuming a roughly spherical body)

$$\rho \approx M/\left(4/3\pi R^3\right),$$

which can be solved for $R$ to give

$$R \approx M^{1/3}\rho^{-1/3}.$$

From now on I will ignore factors like the $4\pi/3$ in the density equation, since our answers are only going to be rough order-of-magnitude approximations anyway. If we put this into our first equation and solve for $M$ we find

$$M \approx \left(\frac{kT}{Gm}\right)^{3/2}\rho^{-1/2}, \qquad (8.9)$$

again dropping simple numerical factors.

Now, Solar System bodies typically have the density of rocks, about $\rho = 6000\,\mathrm{kg\,m^{-3}}$. Let us take the molecule to be $SiO_2$, the main constituent of sand. The silicon **nucleus** contains 14 protons and 14 neutrons, and each oxygen nucleus contains 8 protons and 8 neutrons. Altogether, there are 60 protons and neutrons in one molecule. The mass of the molecule is about 60 times the mass $m_p$ of a proton. (We neglect the small mass difference between protons and neutrons, and we neglect the mass of the electrons, which are a fraction of a percent of the mass of the nuclear particles.) Looking up the mass $m_p$ in Appendix 27, we find that the mass of the molecule is $m = 1 \times 10^{-25}$ kg. Finally, the melting temperature of silicon dioxide is about 2000 K. Putting all these into Equation 8.9, we find the minimum mass of a round body in the Solar System to be about

$$M \approx 3 \times 10^{21}\ \mathrm{kg}. \qquad (8.10)$$

This is about 5% of the mass of the Moon, and much larger than the mass of any known asteroid or comet.

**Exercise 8.3.1:** ***Rounding off the Moon***

Do the algebra that leads to Equation 8.9 from the two equations that precede it. Then put the given numbers into the formula to arrive at Equation 8.10.

rocks. For icy bodies, the mass would be a bit smaller. We should treat this as an order-of-magnitude estimate of the smallest mass of a round astronomical body.

For comparison, the mass of the Moon is $7.3 \times 10^{22}$ kg, so its round shape is no surprise. Our minimum "round" mass is much larger than the mass of any known asteroid or comet, so we should expect asteroids and comets to have irregular shapes, as indeed they all do. In fact, many planetary moons in the Solar System are of smaller mass. For example, Mars' moon Phobos has a mass of $10^{16}$ kg and is very irregular, as Figure 8.2 shows. In fact, the largest irregular body in the Solar System is Saturn's moon Hyperion, whose mass is $1.8 \times 10^{19}$ kg. So our rough calculation is not bad.

### *The Sun is one big atmosphere*

**In this section:** the structure of the Sun is described by the same basic equations as we used to determine the structure of the Earth's atmosphere.

Because the Sun is a balance between gravity and pressure, it is like one giant atmosphere. All the discussion of Chapter 7 can be directly applied here to help us understand the Sun's structure. We will extend the computer program of that chapter to help us make a numerical model of the Sun, and in the next chapter we will apply it to building other stars as well.

What changes do we need to make to apply our atmosphere program to the Sun? One obvious one is that the atmosphere program assumed we were dealing with the gravity of the Earth, not of the Sun. Changing this is not just a matter of changing the value of $g$, the acceleration due to gravity. For an atmosphere, which is a thin layer sitting on top of a big planet, one can assume without losing too much accuracy that the acceleration due to gravity is the same everywhere in the atmosphere. This assumption does not work for the Sun. For example, at the center of the Sun the acceleration must be zero, since particles are being pulled by the different parts of