Investigation 6.3. ***The change in the potential energy***

Here we perform a short calculation to find the way the potential energy changes when there is a small change in the distance of a planet from the Sun. The potential energy at the new position $r - \delta r$ can be manipulated with a little algebra into a form that makes it easy to approximate:

$$-\frac{GmM_\odot}{r-\delta r} = -\frac{GmM_\odot}{r(1-\delta r/r)} = -\frac{GmM_\odot}{r}\left(1-\frac{\delta r}{r}\right)^{-1}.$$

In this last form of the expression we can use Equation 5.2 on page 43 to approximate the last factor:

$$\left(1-\frac{\delta r}{r}\right)^{-1} \approx 1+\frac{\delta r}{r}.$$

When we put this into the expression for the potential energy in the previous equation we find

$$-\frac{GmM_\odot}{r-\delta r} \approx -\frac{GmM_\odot}{r} - \frac{GmM_\odot}{r^2}\delta r.$$

This is the expression we use in Equation 6.18 ‘.

Exercise 6.3.1: ***Changes in potential energy***

Justify (or fill in) the algebraic steps that lead from one term to the next in the first equation in this investigation.

to this equation, since she does not change her position. However, no doubt she still expects to get paid for what she does at the desk!

Time and energy

In this section: we make a fundamental and deep connection between energy conservation and time-invariance of the laws of physics.

We began this chapter by learning how important the law of conservation of energy is. Then we seemed almost to lose the law, in the gravitational slingshot. Of course, energy conservation does still hold in the slingshot, as long as we add together the energies of both bodies. This is not surprising: we should expect that the spacecraft and the planet could exchange energy with each other. But there is another lesson we can draw from this chapter, and that is about the deep relationship between energy and time.

We looked at two kinds of problems: the motion of a body (planet or spacecraft) around the Sun, and the motion of a body around Jupiter. Both were motions under the action of gravity, and in both cases the body that created the gravitational field was too large to be affected by the body. Yet in one case (the Sun), the total energy of the body was constant, and in the other the total energy changed. The only significant difference between the two problems is that in the first case, the Sun was standing still, and in the second Jupiter was moving. That is, in the first case the gravitational field was time-independent, while in the second the field at any given location depended on time (as Jupiter moved past).

We have here a glimpse into one of the most profound relationships in physics: when there is some underlying time-independence in a physical situation, there is usually a conserved energy, and vice versa. The single body moving past Jupiter does not have a conserved energy because it experiences a force field that is time-dependent. But if we consider the body and Jupiter together, then they move in the background field of the Sun, which is time-independent, so their total energy is conserved.

> All the fundamental forces in physics, such as the electric force, gravity, and the nuclear force, work in such a way that the total energy of a collection of bodies is conserved provided that any forces on the bodies from outside the collection are independent of time at any one location.

Essentially, energy is conserved for these bodies if all the rest of the Universe is time-independent. When we come to consider **cosmology** – the study of the Universe as a whole – and the observed expansion of the Universe, we will see how we lose the law of energy conservation: as the Universe expands, its energy simply disappears.

What about other conservation laws? We have also met conservation of angular momentum and of ordinary momentum. Both of these, as well, are associated with some kind of "independence". In the case of angular momentum, it is angular independence: the angular momentum of a planet is constant on its trajectory because the gravitational field of the Sun is independent of the planet's angular position around the Sun. The Sun is spherically symmetric, and this leads to conservation of angular momentum. Ordinary momentum is sometimes called linear momentum because it is conserved in situations where the external forces are constant along straight lines. The ordinary momentum of a planet is certainly not constant along its orbit, and this is because the Sun's gravitational field is not constant in any fixed direction. But when, for example, billiard balls collide on a billiard table, the effect of the Sun's (or indeed the Earth's) gravity is unimportant, and the table itself is flat in any horizontal direction, so the total momentum of the balls in the collision is constant.

Physicists and mathematicians have a name for the general concept that includes time-independence and angular independence. They call it **invariance**. They say that the gravitational field of the Sun is invariant under a change of time (from, say, now to tomorrow), and it is also invariant under a change of angular position. The relation between conservation laws and invariance is something that physicists believe is built into the laws of physics at their deepest level. In fact, physicists today who work on discovering the laws of physics at the highest energies imitate this principle by looking for more abstract kinds of invariances. The approach has been successful so far. Such theories of physics are called *gauge theories*, and all theories of the twentieth century that have unified the nuclear, electromagnetic, and weak forces are gauge theories (see Chapter 27). The principle of invariance is one of the deepest in physics.