we find that the flux of energy carried by a gravitational wave can easily be larger than the flux of light energy we receive from a full Moon. Considering that the source of the wave could be in the Virgo Cluster of galaxies, while the Moon is by comparison right next door, it is clear that the emission of gravitational radiation by an astronomical object can be a catastrophic event, carrying away huge amounts of energy.

▷The weakness of the influence of the gravitational wave on the Earth shows that little of the energy carried by the wave is left in a detector. This is due to the weakness of gravity itself, not to any lack of energy in the waves.

Because the equivalence principle allows us to wipe out any gravitational field locally, even a gravitational wave, the energy of a wave is really only well-defined as an *average* over a region of space whose size is larger than the wavelength of the wave, and over a time longer than the period of the wave. Extended bodies can therefore only extract the energy if they interact with the wave over a long enough time or a large enough distance.

In the present case, the geometry of spacetime is constantly changing because of the gravitational wave, so energy conservation needs to be treated carefully. Indeed, if we consider just a matter system (such as a detector for gravitational waves), then the waves are an external time-dependent influence on it, and we do not expect its energy to be constant. That is good: one hopes a wave will disturb the detector enough to allow us to measure it! To arrive at a conserved energy that can be exchanged between the detector and the wave, we have to treat the wave and detector together. This is not so easy in general relativity, because it is not easy to define the wave separately from the rest of the geometry.

Figure 22.2. *Joseph Taylor. (Photograph by Robert Matthew provided courtesy Princeton University.)*

To see the reason for this, consider water waves. Drop a rubber duck into the still water in a bathtub. Waves ripple out from the place where it lands. We have no trouble distinguishing the waves from the rest of the water, and eventually the waves disappear and we return to the same still water surface as before. By contrast, look at a stormy ocean during, say, a hurricane. Near the beach, what are the waves? Sometimes there is water, sometimes beach. The whole ocean is moving. There is no way to define waves as a disturbance *on* the water.

Strong, time-dependent gravitational fields must be treated with more care in general relativity than we are able to do here. Recall that we learned in Chapter 6 that energy is only conserved in situations where external forces are independent of time. For weak waves, it is possible to define their energy with reference to the "background" or undisturbed geometry, which is there before the wave arrives and after it passes. But if the geometry is strongly distorted, the distinction between wave and background has little meaning. In such cases, physicists do not speak about waves. They only speak of the time-dependent geometry. But normally such regions are small, and outside of them the waves take shape as they move away.

Figure 22.3. *Russell Hulse. (Photograph provided courtesy Princeton Plasma Physics Laboratory.)*

The Binary Pulsar: a Nobel-Prize laboratory

In this section: the discovery of the first pulsar in a binary system provided the first experimental confirmation of the theory of gravitational radiation. It has become a test of extraordinary accuracy.

In 1974, two astronomers made a discovery that was finally to give gravitational radiation theory an experimental foundation. The American radio astronomer Joseph H Taylor (b. 1941) had sent his graduate student, Russell Hulse (b. 1950), to observe pulsars with the largest radio telescope in the world, the Arecibo telescope in Puerto Rico. Hulse noticed a signal that appeared to be a pulsar, but strangely its pulse frequency kept changing. He told Taylor, who soon joined him at Arecibo, and together they determined that the pulsar was changing its frequency in a periodic way, coming back to its original frequency every eight hours or so. For a star like a neutron star to change its rotation speed that rapidly seemed impossible, like trying to slow a thundering train. Something else had to be making the frequency change. The conclusion was inescapable: the pulsar was in orbit around another star, with a period of eight hours, and the change in the frequency was simply the Doppler effect as the pulsar went away and came back again in its orbit.

But an 8 h period is extraordinarily short. No binary had ever before been observed with such a short period. Mercury goes around the Sun in 88 days. A satellite of the Sun would have to be just skimming its surface in order to have an orbital period as small as that. But the pulsar was not skimming a star: there was no evidence of friction making the orbit change quickly, and later optical observations of the pulsar's position did not reveal any star, not even a white dwarf. The pulsar, therefore, was orbiting another neutron star or a black hole. Whatever is there radiates nothing we can see. Because this was the first pulsar discovered in a binary system, astronomers began to call it *the* Binary Pulsar. Radio astronomers have subsequently discovered many other pulsars in binaries, so the name is no longer a good one. We shall call it the Hulse–Taylor binary pulsar or simply PSR1913+16.

The orbit of PSR1913+16 is highly relativistic, its speed being about 0.1% of the speed of light. The orbit is, fortunately, a rather eccentric ellipse, so the precession of the perihelion (in this case, it is called the **periastron**) is easy to measure because it is 4° per year. (Compare this to Mercury, where one waits a century or so for the effect to build up enough to measure it accurately!) As we saw in Chapter 18, the precession depends on the mass of the companion, but when (as is the case here) the satellite's mass is not negligible compared to the companion, it is not possible to determine each mass individually from the precession alone.

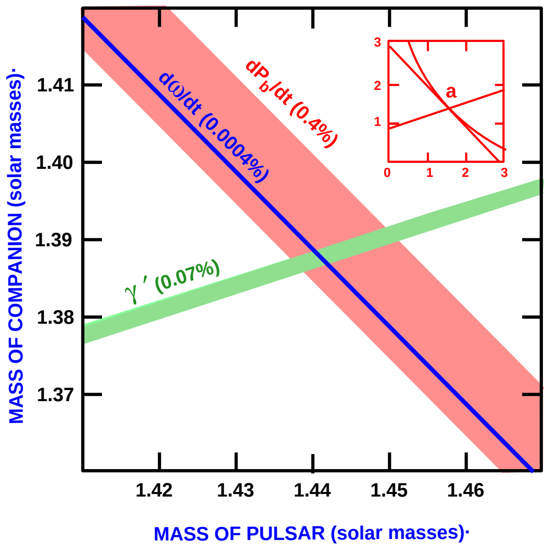

__Figure 22.4.__ This figure shows the way the masses of the stars in the Hulse–Taylor pulsar system are determined, and how the observed period decrease is consistent with them. The axes are the masses of the two stars, and the lines show how the observed properties of the system depend on the masses. The line labeled γ' is the combined redshift and time-delay term. Any combination of stellar masses on this line would give the observed delay. The width of the line indicates the spread of values allowed by the observations. The extremely narrow line labeled $d\omega/dt$ is the region allowed by measurements of the periastron shift of the elliptical orbit. The narrowness of this line shows how well this is determined. The broader area around this line, labeled dP_b/dt, is the region allowed by the observed shortening of the orbital period. The fact that all three strips overlap in one region (at masses about 1.39 and 1.44 times the mass of the Sun) is a strong test of general relativity. In another theory of gravity, they need not coincide. The inset figure is the same figure drawn with a larger range of masses. This shows that the curve for the orbital period bends away from the periastron curve over a larger region; if general relativity were not correct then these two curves might not touch at all. Figure courtesy of C M Will.

But Taylor, by repeated careful observations spread over many months, was able to extract another relativistic effect. He could see the change in the pulsar's spin rate as it moved closer to and further from its companion. As we discussed in Chapter 20, this is caused by two effects that act together. The first is the changing gravitational redshift as the pulsar moves in and out in the companion's gravitational field; this redshift affects the spin rate in the same way it would any other clock. The second is the bending of the path of the radio waves as they pass near the companion, which introduces a changing time-delay that adds to the gravitational redshift. The combination of these two effects and the precession allowed Taylor to deduce the masses of both stars, as shown in Figure 22.4. Remarkably, they are both of mass about $1.4M_\odot$. Today the masses are known to an accuracy of better than 0.1%, the best mass determinations of any objects outside the Solar System.

Because the companion has a mass in the range of masses of neutron stars, it seems unlikely it could be a black hole: pressure would have halted its collapse. So it is assumed to be another neutron star, but there are no direct observations of it, no pulses of radiation or faint glow of X-rays that might confirm this.

The Hulse–Taylor pulsar is a laboratory for relativity. It confirms the perihelion precession calculated by Einstein to much higher accuracy than Mercury does. It demonstrates the gravitational redshift of a huge clock, showing that the equivalence principle works even for timekeeping by the spin of relativistic stars. All this information is enough to

Example system	Component mass M	Orbit radius R	Distance r	$f_{\rm gw}$ (Hz)	$t_{\rm gw}$	h	$L_{\rm gw}$ ($L_\odot$)
Hulse–Taylor	$1.4M_\odot$	1×10^6 km	8 kpc	6.9×10^{-5}	7.4×10^9 y	3.5×10^{-23}	1.5×10^{-3}
NS-NS	$1.4M_\odot$	50 km	200 Mpc	190	1.5 s	2.8×10^{-23}	4.7×10^{18}
MBH–MBH	$1.4 \times 10^6 M_\odot$	5×10^7 km	4 Gpc	1.9×10^{-4}	1.5×10^6 s	1.4×10^{-18}	4.7×10^{18}

Table 22.1. *Three binary systems of the type that could be detected by ground-based or space-based gravitational wave detectors. For simplicity the systems are assumed to contain equal-mass components in a circular orbit around one another. For each example we specify the masses of the stars, the orbital radius, and the system's distance from us; then we calculate the frequency of the gravitational waves f_{gw} from Equation 22.6 on the following page, the chirp time t_{gw} (orbital shrinking time-scale due to gravitational waves) from Equation 22.12 on page 321, the maximum gravitational wave amplitude h at the Earth from Equation 22.7 on the following page, and the gravitational wave luminosity L_{gw} from Equation 22.10 on the next page. The latter is given in units of the solar luminosity $L_\odot$. For the system in the first line, which is a circular-orbit version of the Hulse–Taylor binary pulsar system, the calculated chirp time is longer than the observed one by a factor of about 12, because of the eccentricity of the real orbit. This brings the stars closer together for a fraction of their orbits, and so the average value of the luminosity is larger. The second and third systems are binaries that have the same compactness, as measured by GM/Rc^2. Notice that they have the same luminosity, despite having very different masses. The more massive system (third line) has a longer lifetime, allowing it to radiate more energy in total. The third system also has the strongest amplitude despite being at a very great distance, where the cosmological expansion redshift is about one.*

tell us everything we would want to know about the orbit.

And on top of all of this, the orbit shrinks. As gravitational waves carry energy away from the orbit, the stars get closer together, and the orbital period decreases. This is exactly the effect Laplace looked for in planetary orbits. General relativity of course provides a prediction for the rate of shrinking, and it has no adjustable numbers in it. Since physicists know the masses and separations of the stars from the other relativistic effects, they can use general relativity to predict exactly how rapidly the period should decrease. We make an estimate of the energy radiated by the system in Investigation 22.2 on the next page, and from it the expected rate of change of the period in Investigation 22.3 on page 321. The prediction is that the period should lose $(2.4427 \pm 0.00005) \times 10^{-12}$ seconds per second. The uncertainty of $\pm 0.00005 \times 10^{-12}$ seconds per second comes from the uncertainties in the deduced masses of the stars. The measurement is that the system is losing $(2.4349 \pm 0.010) \times 10^{-12}$ seconds per second. The uncertainty here is the observational accuracy. The two numbers agree within the uncertainties, as is shown in Figure 22.4.

This is a stringent test of general relativity and a striking confirmation of the predictions of the theory regarding gravitational radiation. For their discovery of this immensely important system, Hulse and Taylor received the Nobel Prize for Physics in 1993. Unlike the case of Jocelyn Bell, to which we referred in Chapter 20, in this case the Nobel committee included the graduate student who first recognized the phenomenon. Perhaps the controversy over Bell's omission was a lesson learned by that committee.

▷The shrinking of the orbit happens because general relativity creates a small *gravitational radiation reaction* force, so named because it is the reaction of the orbit to the loss of energy to gravitational waves. We mentioned this in Chapter 2.

Gravitational waves from binary systems

In this section: there is a wide variety of binary systems that could be radiating detectable gravitational waves. Coalescing neutron star and black hole binaries are among the most important targets of ground-based detectors, and a detector in space could obtain important information about a large variety of massive binaries.

Although the Hulse–Taylor binary system is radiating gravitational waves with a strength that physicists can compute exactly, there is little hope of directly detecting them in the near future: their frequency (given in Table 22.1) is too low for detectors now being planned, as we discuss later. Nevertheless, other binary systems are the most important gravitational wave sources that the detectors now planned or under construction will search for.

Astronomers now know that there are many other binaries with even shorter periods than the Hulse–Taylor system. A few systems that are known from optical or X-ray observations in our Galaxy have periods that will be detectable by the space-based detector LISA, which we will describe at the end of this chapter. Even