Preface

From the author to the reader

Why this book is about gravity

During the 30 years that I have done research in gravitation, I have watched with amazement and delight as my colleagues in astronomy have, step-by-step, opened up almost the entire Universe to our view. And what a view! There are punctures in space called black holes that capture gas and stars with a relentless and unbreakable grip; there are 10 km balls called neutron stars that are immense overgrown atomic nuclei with more mass than our Sun, that spin about their axes hundreds of times per second while emitting intense beams of radiation; there are bursts of gamma-rays from the most remote regions of the Universe that are so intense that they outshine the rest of the Universe for a short time; and most strikingly of all there was the beginning of time itself in an explosion of pure energy, driven by a force we do not understand, in which matter as we know it did not exist, in which even the laws of Nature themselves were mutable.

This astonishing Universe has captured the imagination of many people, among them many scientists. Physicists trained in a number of disciplines have applied themselves to explaining these and many more less spectacular but equally important phenomena, such as: how the chemical elements were made; where stars come from and how they evolve and die; how the vast systems of stars called galaxies formed and why they have grouped themselves into clumps and long chains; why a Universe filled with bright stars seems to contain even more matter that cannot form stars – and so remains dark.

From all this scientific activity has come a great deal of understanding. We know not just *what* happens, but in many cases *how* and *why.* Physicists, astrophysicists, and astronomers have been able to put together a coherent story of how our Universe began and of how its immense variety evolved.

▷The link between gravity and the wonders of astronomy goes right back to Galileo, who founded the science of gravity. Using a telescope for the first time, Galileo became the first person to understand that the Milky Way is composed of stars, that Venus shines by light reflected from the Sun, that the Sun is plagued by spots, that Jupiter holds its own satellites in orbit around itself in imitation of the Solar System. Our amazement at astronomers' discoveries today helps us to appreciate what Galileo's contemporaries must have felt at his.

The central theme of the story of the Universe turns out to be *gravity.*

Gravity is the one force of Nature that operates everywhere; it controls the effects of all the other forces wherever they act; it regulates countless natural clocks, from the orbits of planets to the lifetimes of stars. Gravity rules the most violent places in the Universe – quasars, pulsars, gamma-ray bursters, supernovae – and the most quiet – black holes, molecular clouds, the cosmic microwave background radiation. Today gravity binds stars and galaxies and clusters of galaxies together, but much earlier it pushed the Universe violently apart. Gravity explains the uniformity of the Universe on very large scales and its incredible variety on small scales. Gravity even laid the path toward the evolution of life itself. If we understand how gravity works, then we begin to understand the Universe.

Rich as our understanding of the workings of gravity in the Universe has become, it is far from complete. The gaps are not just hidden regions, phenomena yet to be discovered, although when such discoveries occur they are sure to bring more amazement and delight. The most exciting gaps are those in our understanding of the laws of Nature.

▷The word "revolution" gets used so much these days in discussions of progress in the sciences, that I hesitate to use it here. But I know no better word.

The enormous advances in astronomy that I have witnessed in my working life have brought us to the threshold of a profound revolution in our understanding of gravity itself. Many physicists today are working to unify gravity with the other forces of Nature, which will lead to what is called a quantum theory of gravity. There are aspects of the Universe that will not be explained without this new theory, and there are clues to the new theory in many of the currently unexplained puzzles of the Universe.

▷Our tour will be thought-provoking, sometimes demanding, even laborious. But we will not leave you, the reader, behind. If you start with high-school mathematics skills – see the section "How this book uses mathematics" beginning on the next page – then you will be able to follow the discussion all the way. And if you put in the effort to study and run the computer programs that allow you to study areas where simple mathematics does not suffice, then you can reach real expertise in some areas. Our goal is ambitious, but I hope you find it worth the effort!

> This book is about gravity at the threshold of this revolution. We will take a tour of the Universe from the ground up. We will start at the surface of the Earth and move outwards through the Solar System, the Galaxy, and beyond to a scale where our Galaxy is the merest atom in the corpus of the Universe.
>
> We will learn about gravity and the other laws that govern the Universe, first as understood by Newton and his successors, then as understood by Einstein and modern physicists. We will use these laws to see how the parts of the Universe work, how they relate to one another, and how they may have come to be. By the end of our tour we will see the Universe and its physical laws, not merely as a collection of fascinating but separate phenomena, but rather as a unity.
>
> Our goal is not just to wonder and marvel at our Universe, nor simply to admire the cleverness of the scientists who have made the Universe at least partially understandable. Instead, *our goal is to understand* how *the Universe works, to begin to* think *about the Universe in the same way that these scientists themselves do.*

How gravity evolves

Gravity, the oldest force known to mankind, is in many ways also the youngest. It is understood well enough to explain stars, black holes and the Big Bang, and yet in some ways it is not understood at all. Explaining gravity required the two greatest scientific minds of modern history, Isaac Newton and Albert Einstein; and now hundreds of the brightest theoretical physicists are working to invent it once again. Each time gravity has been re-invented, it has sparked a revolution. Newton's theory of gravity stimulated huge advances in mathematics and astronomy; indeed, it was the beginning of modern theoretical physics. Einstein's theory of gravity, which he called general relativity, opened up completely unexpected phenomena to investigation: black holes, gravitational waves, the Big Bang. When, sometime in the future, gravity changes into quantum gravity, possibly becoming just one of many faces of a unified theory of all the physical forces, the ensuing revolution may be even more far-reaching.

Each of these revolutions has built on the previous one, without undermining it. Newton's gravity is just as important today for explaining the motions of the planets as in Newton's time. It is used to predict the trajectories of spacecraft and to understand the structure of galaxies. Yet general relativity underpins all of this, because Newton's gravity is only an approximation to the real thing. We need only Newton to help us understand how a star is born and evolves; but when the star's evolution leads to gravitational collapse and a supernova, then we have to ask Einstein's help to understand the neutron star or black hole that is left behind. When we have a theory of quantum gravity, it won't stop us from using general relativity to explain how the Universe expanded after the Big Bang; but if we want to know where the Big Bang came from, and why (or whether) time itself started just then, we will need to ask the quantum theory.

There is a deeper reason for this continuity from one revolution to the next. As an example, consider the fact that two of the fundamental ideas in Einstein's general relativity, called the principle of relativity and the principle of equivalence, originated with Galileo. Einstein's revolution brought a complete change in the mathematical form of the theory, added new ideas, and opened up new phenomena to investigation. But there was a profound continuity in physical ideas, and these were as important to Einstein as the mathematical form of the theory. The coming quantum revolution will surely likewise be grounded firmly in concepts that physicists today use to understand gravity. These physical ideas are the subject of this book.

Of course, this book deals mainly with what we already know about the role that gravity plays in the Universe, which is the result of the first two revolutions. You, the reader, will learn what Newton's gravity is, and how it regulates planets, stars, and galaxies. You will learn what relativity means, and how general relativity leads to black holes and the Big Bang. But I want you also to be able to follow the continuity of ideas, to see for example how Newtonian gravity prepared the way for relativity. In the earliest chapters we will see that Newtonian gravity already contained half of relativity, that it contained the equivalence principle that guided Einstein to general relativity, even that it foresaw the existence of black holes and gravitational lenses. In the same way, general relativity contains seeds that will blossom only when the third revolution arrives, such as why the theory allows the cosmological constant. I will try to point out some of these seeds as we go along, usually by asking questions that general relativity or modern astronomy suggests but does not answer. The final chapter is devoted entirely to such questions.

▷Chapter 1
▷Chapter 2
▷Chapter 4
▷Chapter 27
▷Chapter 27

Why this book is about more than gravity

Because gravity is the dominant force anywhere outside of the surface of the Earth, this book covers a lot of astronomy. But instead of just touring randomly around the Universe, we have a theme: gravity as the engine that makes things happen everywhere. This theme unifies and simplifies the study of astronomy. If we understand gravity on the Earth, then it is easier to understand it in the Solar System. If we understand it in the Solar System, then we have an easier time grasping how it acts in stars and black holes. And so it goes, right up to the largest scales, to the Universe as a whole.

▷Chapters 1–3
▷Chapters 4–8
▷Chapters 9–13, Chapter 21
▷Chapters 24–27

Because gravity usually acts in concert with other forces of physics, studying gravity this way also gives us the opportunity to investigate much of the rest of physics along the way. For example, quantum theory and gas dynamics play important roles in stars, and so we study them in their own right where we need them. Even if you have studied these subjects before, you may be surprised at some of the connections to other parts of physics that you will discover by looking at them in the context of explaining a star.

▷Beginning in Chapter 7

How this book uses mathematics

You may already have guessed that this book is not a "gee-whizz" tour of the Universe: this is a book for people who are not afraid to think, who want to understand what gravity is, who want to go beyond the superficial level of understanding that many popular books settle for. But this is also not an advanced texbook. We shall steer a careful middle course between the over-simplification of some popular treatments and the dense complexity of many advanced mathematical texts.

This book has equations, but the equations use algebra and (a little) trigonometry, not advanced university mathematics. What is required in place of advanced mathematics is thought: readers are asked to reason carefully, to follow the links between subjects. You will find that you can climb the ladder from gravity on the Earth to gravity (and even anti-gravity) in the Universe if you go one step at a time, making sure you place each foot securely and carefully on the rungs as you climb. In return for putting in the thought that this book asks, you can get much further than you might have expected in understanding gravity and its manifestations in astronomy. School students and university undergraduates will find that this book offers them an early avenue into subjects that are usually regarded as much too advanced for them.

There is no calculus in this book, despite the fact that calculus is the workhorse mathematical tool of physics. Wherever possible I have tried to present a physical argument as a substitute for the mathematical one that physicists are used to. This has the great advantage that it makes connections between different parts of physics clearer and the logical reasoning more direct. It has the disadvantage, of course, that it is not always possible to do this: there are places where using more advanced mathematics really is necessary for a pen-and-paper treatment. In such cases I have often turned instead to a computer program. These programs are not "black boxes": their construction is discussed in detail. See the next section for a discussion of why they are good substitutes for advanced mathematics.

Sometimes I have had to resort to that awful phrase "it can be shown with more advanced mathematics" or something like it. I have avoided this whenever I could, but there are times when it seemed to me that any argument I could give for a particular result would be over-simplified, it would hide or corrupt the truth. It is best in such situations to be honest and accept that our mathematical tools at this level are not always sufficient. Our aim is not to cut corners, but always to remain true to the physics.

▷See Figure 19.1 on page 242.

In fact, it is possible to read this book while avoiding most of the equations, if you want to. All the extended algebraic calculations are placed in special boxes, called investigations. These are set aside on a light-gray background. Skipping these boxes might be a good strategy if you are short of time, or on your first reading. If you skip them you will just have to take on faith some of their results, which are then used in the main text. Many of the investigations contain exercises, which offer you a chance to test your understanding. I believe strongly that doing exercises is the most effective way to get comfortable with an important result. If you are using this book as a textbook for a course, then I hope your teacher will expect you to do the exercises!

▷Solutions to the exercises can be found on the book's website. See the next page.

How to go beyond this book by using computers

For those of you who have access to computers and want to use them, I have provided a way to reach the results of some very advanced mathematics by using computer programs that only require the mathematical level of the rest of this book. This is your best way to get to some of the results that algebra alone cannot reach. The programs can be downloaded from the website (see the next section) and used right away – just run them and look at the results. You can then change some of the numbers they work with, for example to compute the orbit of Jupiter instead of Mercury, without looking inside the program. But the way to get the most out of them is to study the investigations in which they are described, look inside the programs, and even experiment with changing the code.

As an example of the power of computer programs, consider the motion of a planet around the Sun. Newton's law of gravity giving the forces that govern the motion of the planet is not hard to write down or to understand using pure algebra.

Using it – solving it – to find the planet's orbit is not so easy with pen and paper. The usual way that university physics students learn how to show, for example, that the orbit is an ellipse is by using some rather sophisticated calculus. They may have to wait until their second or third year to get to this important result. In Chapter 4 we achieve the same thing by writing a simple computer program that moves the particle along in its orbit step-by-step. The law of gravity translates directly into a prescription for algebraic calculations that the computer can readily do. The orbit that comes out is clearly an ellipse. The calculation can be done as accurately as one wishes by simply telling the computer to take smaller steps. Repetitious computations like this are what computers do best.

Even better, a computer program can be modified and used again in situations that would require even more sophisticated calculus to make progress. We modify the orbit program slightly in Chapter 13 to explore what happens when three stars interact with one another, a situation that often results in one of them being expelled from the system at high speed. We modify the orbit program yet again in Chapter 21 to show that orbits of bodies around black holes are not ellipses, but rather make rosette patterns. And finally in Chapter 24 we modify it another time to calculate the expansion of the Universe itself.

▷This so-called three-body problem is a classic problem that cannot be solved by analytic calculus alone. All scientists who study it use computer programs.

The programs are available from the website of this book (see below). They are written in the free and widely available language Java®. Since the popularity of this language is steadily increasing, you may find that following the computer programs here will help you to learn a language that will be useful to you later. In any case, the Java language is not very different from the most popular language of all, C. If you have never programmed before, the package you can download from the website provides an easy way of starting.

Using the glossary and website

There are two aids for you to get more information than is in the text: the glossary, which begins on page 421, and the website. The glossary contains definitions of many of the terms used in this book. Some of the terms in the glossary are not defined in the book because I have assumed that most readers will know what they mean, or because they are not central to the subject matter. Other terms in the glossary are important words that are explained somewhere in the book but which are placed in the glossary so that you can conveniently look them up when you encounter them again. All terms in the glossary are printed in **boldface** type when they are first used in the book. I don't use boldface for any other reason, so whenever you see it you will know that it marks an entry in the glossary.

▷Terms printed in **boldface type** in the text are contained in the glossary.

The website for this book is

`http://www.gravityfromthegroundup.org`

It contains

- the Java programs for you to download;
- a free version of the Triana® software environment for running the programs and displaying their results graphically;
- solutions of all the exercises;
- links to allow you to download and install Java and other programs needed for your computer;
- additional illustrations for some of the chapters;
- a way of submitting comments, misprints you have found, or suggestions that could be incorporated in future editions; and

- links to useful websites where you can follow up some of the material covered in the book.

Visit the website: it is a valuable addition to this book, and it is completely free.

◊◊◊◊

From the author to his colleagues

Teaching gravitation

Although this book is aimed at beginners, many readers may be my colleagues, professional physicists who may be using the book in a course they are teaching, supervising a student who is using the book for a self-study program, or just looking for a different point of view on the subject. For such readers, this section enlarges on the pedagogical side of my approach to this subject.

The aim of this book is to introduce gravitation theory as a unified subject, and especially to show the key role that gravitation plays in the phenomena of the Universe. An associated pedagogical goal is to develop the reader's ability to think physically by using physical reasoning rather than advanced mathematics to move through the subject. The restriction to elementary mathematics presents real challenges in presentation, but it allows one to treat the entire theory in a unified way, from Newton to Einstein and beyond ... from the ground up, in other words.

Mathematics is not just a powerful tool in physics, it is the *reference language* of science: it is the language in which the fundamental theories are written, the medium which is used to deduce the predictions from a theory. But physicists generally supplement mathematical deduction with physical reasoning. Indeed, physicists who can do this reliably are widely admired for their great "physical intuition". When physicists are inventing new theories, searching for new physics, or trying to explain phenomena not previously encountered, physical reasoning often leads and mathematical reasoning follows: new ideas suggested first by physical arguments are then put into mathematical form and tested for consistency and suitability. Yet when physicists teach known physics to newcomers, the balance more often falls heavily toward mathematical reasoning, the reference form of the theory. Students need of course to master the mathematical form of a theory in order to be able to work seriously with it or to go beyond it, and physics teaching generally focuses on that requirement.

But I believe that this focus is often too narrow. It is important to remind ourselves that there is usually a line of physical reasoning that moves along parallel to the mathematical. Ideally, each way of thinking supports the other. But for students with unsophisticated mathematical tools, it should be possible to make significant progress using mainly physical reasoning. After all, if the principal theories of physics were invented by using physical arguments as guides, then it should be possible to teach important things about those theories in the same way.

Putting mathematical presentation first has another undesirable pedagogical side-effect: it is customary to teach some physical theories in discontinuous segments in order to allow students time to learn more sophisticated mathematics in between. Nowhere is this more arresting than for gravitation theory. Newton's law of gravity is presented in high school, but even using it to find the simple elliptical orbit of a planet must wait until the student masters integral calculus, in the first or second year of a university course. Because of its use of tensors and differential geometry, general relativity has to wait until the final undergraduate year at the earliest; most physicists encounter it first as graduate students, if at all.

Yet there are very good reasons for teaching gravitation theory as a unified subject. The continuity of physical ideas and phenomena is strong. Consider the following sampling, which is by no means exhaustive.

- The equivalence principle and the principle of relativity – so important to Einstein – originated with Galileo.
- Most physicists find it remarkable that black holes and the gravitational deflection of light were discussed by scientists more than a century before Einstein (see Chapter 2). Yet surely this simply means that the links between Newtonian and Einsteinian gravity go deeper than most of us assume.
- There are more similarities than differences between Newtonian and relativistic stars. Even the gravitational effects caused by the spins of stars and black holes have their roots in Newtonian gravity (Chapter 19).
- If we want to trace the histories of the objects in Newton's universe, such as planets and people, all the way back to their ultimate roots, we inevitably encounter the hot, slightly lumpy plasma that we call the Big Bang. The inevitability of the Big Bang has as much to do with the argument of the eighteenth-century physicist Olbers, who understood how strange it is that the sky is dark at night, as it has with Einstein.
- And even Einstein's theory of general relativity itself, for all its mathematical complexity, is arguably as close in physical content to Newton's as it was possible for Einstein to make it while still respecting special relativity.

To teach the broad sweep of gravity as a unified whole to an audience that normally only gets taught about circular planetary orbits, I have followed the pedagogical philosphy outlined earlier: using the minimum level of mathematical sophistication, I have tried to progress through gravitation theory as much as possible by using physical arguments. This started out, quite frankly, as an experiment, a challenge to myself, and I have learned much from it, especially about the connections between and continuity of ideas in this subject. For example, it is satisfying that it is natural to introduce both the principles of relativity and of equivalence in Chapter 1, followed immediately by the gravitational effect on time in Chapter 2, without ever leaving the vicinity of the Earth, before even considering Newton's law of gravitation. When these principles turn up again in special and general relativity, they are old friends. When I explain in Chapter 19 that gravity in Einstein's picture is found mainly in the curvature of time, it is not hard to justify this from the discussion in Chapter 2. It is equally satisfying to calculate the Newtonian gravitational force exerted by a spherical body (Chapter 4) – using one of the computer programs to do the integral calculus – and then to find that one needs nothing more than this to calculate the evolution of a homogeneous and isotropic cosmological model (Chapter 24). It is fascinating to calculate the fundamental normal mode frequency of the Sun (Chapter 8) in Newtonian gravity and then to find that the same formula comes within a factor of two of the right answer for the pulsations of a disturbed black hole (Chapter 21). It is equally fascinating to discover that one can derive the Lense–Thirring effect quantitatively from Newtonian gravity and special relativity only if one uses the Einstein form of the active gravitational mass as the source of gravity (Chapter 19), and thereby to establish deep links between Newtonian gravity, the spinning black holes (Chapter 21), and the inflationary universe (Chapter 24). The list could be much longer.

This approach to teaching gravitation will surely not appeal to everyone, but I hope that especially my scientific colleagues in relativity will find it amusing to see how many threads continue from Newtonian gravity to relativity, how many apparently abstract and mathematical properties of relativistic gravity have clear and simple physical derivations, and how much easier it is to introduce general relativity if Newtonian gravity has been taught in a way that emphasizes the ideas that continue into relativity.

Guiding students through this book

The book can be used by teachers for guided self-study, as a textbook for a course on physics or astronomy for non-scientists, or as a main or supplementary text in conventional university physics and astronomy courses.

Courses for non-scientists need to excite and challenge students without overwhelming them with mathematics. With its emphasis on developing physical intuition, this book aims directly at what is probably the most important goal of such a course: students should learn what it means to think like a physicist. The exercises can play an important role. Depending on the length of such a course and the background of the students in it, the teacher may want to be selective in what material to focus on. I would welcome feedback (via the website) from anyone teaching such a course.

When using this book with physics or astronomy undergraduates, the obvious problem is that the book has a "vertical" integration: it covers material that is usually treated in different courses in different years. Indeed, that is why I have written it. It can be helpful for beginners to expose them to some of these advanced ideas and to let them explore them with the aid of the computer programs before they reach the mathematical level needed to treat them in the conventional way, later in their education. Again I would welcome feedback via the website from lecturers who use this book in such courses, either as the main or as a supplementary text.

The computer programs deserve special attention from the teacher. They fill the gaps between algebra and calculus for beginners, while for students who continue to study physics they are good preparation for later analytical attacks on problems.

Let me give an example. Consider the computer program for finding the motion of a planet around the Sun, to which I referred earlier. The mathematical way that undergraduate physics students learn that its orbit is an ellipse is by writing Newton's law of motion as a differential equation and solving it using fairly sophisticated calculus. They often have to wait a year or two in their undergraduate course before they have the skill to do this. The solution can instead be found using a computer if we replace the differential equations with finite difference equations. By formulating Newton's law from the start in terms of finite differences – the change in velocity in a small but finite time-interval is approximately the acceleration at the beginning of the time-interval times the time-interval – we have an immediate entry into the computer simulation. The formulation is obvious to students, and just as obvious is the idea that if one makes the time-step smaller and smaller then the computer solution becomes a better and better approximation to the real thing. This is calculus in practice, and if students meet calculus later in their mathematics education, then they know they have already been doing it on the computer.

▷Besides the four equations-of-motion computer problems mentioned earlier, the book applies computers to a variety of other problems. Students can prove that the gravitational field outside a spherical body is the same as if all its mass were concentrated at its center, by adding up the forces from small elements of the body. They can make a computer model of the Earth's atmosphere, and later use the same program to model the Sun and a neutron star. In each case the problems are formulated from the start in terms of small differences rather than derivatives. And there are no compromises: we don't have to over-simplify these problems in order to put them on the computer.

Computers are already used in this way in many introductory physics courses. Some of the best use spreadsheets, because they are widely available, they contain all the required mathematical operations, and they can display results as graphs. For this book, however, I have chosen instead to use the programming language Java®. It is also widely available, free, and mathematically complete. The Triana® environment that can be downloaded for free from the website provides the ability to run programs as black boxes and get graphical output. I prefer the fact that Java programs are closer in structure to those that students may write later in their careers (in Fortran, C, or even Java). But lecturers who already use spreadsheets with their students should have little trouble transferring the programs to that format.

Acknowledgements

This book has taken many years to write, and in that time I have learned much from many colleagues in astronomy, physics, and relativity. Some have contributed directly to this book with constructive criticism, creative input, or tracking down resources and historical material; others have simply taught me things I did not know. I would especially like to acknowledge my indebtedness to Robert Beig, Werner Benger, Jiří Bičák, Curt Cutler, Thibault Damour, Karsten Danzmann, Mike Edmunds, Jürgen Ehlers, Jim Hartle, Günther Hasinger, Jim Hough, Klaus Fricke, Matthew Griffiths, Geraint Lewis, Elke Müller, Charlie Misner, Jürgen Renn, Rachel Schutz, Ed Seidel, Kip Thorne, Joachim Wambsganss, Ant Whitworth, Chandra Wickramasinghe, and Cliff Will. None of them, of course, bears responsibility for any errors that remain in the book. My employers, the Max Planck Society and Cardiff University, have kindly made it possible for me on a number of occasions to to get away from my normal duties and write. My editors at Cambridge University Press – Simon Mitton, Rufus Neal, Simon Capelin, Tamsin van Essen – deserve special thanks for their patience and encouragement while waiting for a manuscript that must have seemed like it might never arrive, and which kept growing well beyond its original planned length. Fiona Chapman of Cambridge University Press helped enormously with the presentation. And last but by no means least, I want to thank my family, especially Siân, for putting up with my hiding away to write on countless evenings, weekends, and holidays, and for their unwavering belief that the final result would be worth it.

Ready to start

This preface is long enough. Beginners, or rusty old-timers, should check the review of background material that follows next; others should jump straight to Chapter 1. I wish the reader a satisfying and enlightening journey through the universe of gravity, from the ground up.

Bernard Schutz
Golm, Germany
16 February 2003