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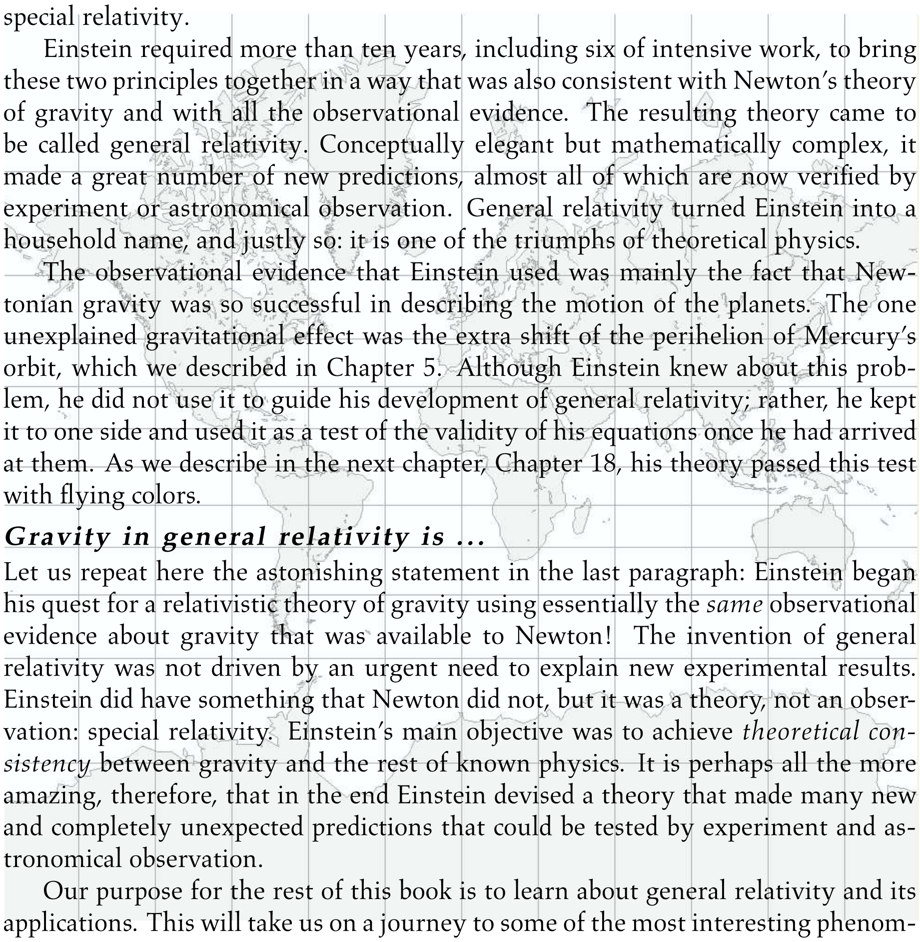
Spacetime geometry:
finding out what is *not* relative

In this chapter: we take our first steps toward understanding general relativity by describing special relativity in terms of the geometry of four-dimensional spacetime. This geometry describes in an elegant and visual way the algebraic predictions of special relativity that we met in the previous chapters. The geometry of special relativity is flat, and we learn how the equivalence principle will allow us to curve it up and produce gravity.

When Einstein began to develop his theory of gravity, he knew he had to build on special relativity, but he felt strongly that he also had to preserve Galileo's other great contribution to physics, the principle of equivalence (Chapter 1). As with special relativity, Einstein worked by blending the old and the new in equal proportions: special relativity combined the old principle of relativity with the new principle of the universality of the speed of light; in his new theory of gravity Einstein combined the old principle of equivalence with his new theory of special relativity.



▷Underneath the text on this page is the familiar Mercator projection map of the entire Earth. This map illustrates strikingly the fact that the surface of the Earth cannot be represented faithfully on flat paper. The Earth is curved, and mapping it flat distorts distances. In this case, the distances near the poles are exaggeratedly large.

Einstein required more than ten years, including six of intensive work, to bring these two principles together in a way that was also consistent with Newton's theory of gravity and with all the observational evidence. The resulting theory came to be called general relativity. Conceptually elegant but mathematically complex, it made a great number of new predictions, almost all of which are now verified by experiment or astronomical observation. General relativity turned Einstein into a household name, and justly so: it is one of the triumphs of theoretical physics.

The observational evidence that Einstein used was mainly the fact that Newtonian gravity was so successful in describing the motion of the planets. The one unexplained gravitational effect was the extra shift of the perihelion of Mercury's orbit, which we described in Chapter 5. Although Einstein knew about this problem, he did not use it to guide his development of general relativity; rather, he kept it to one side and used it as a test of the validity of his equations once he had arrived at them. As we describe in the next chapter, Chapter 18, his theory passed this test with flying colors.

Gravity in general relativity is ...

In this section: we look ahead at the ways we will learn to use general relativity in the rest of this book.

Let us repeat here the astonishing statement in the last paragraph: Einstein began his quest for a relativistic theory of gravity using essentially the *same* observational evidence about gravity that was available to Newton! The invention of general relativity was not driven by an urgent need to explain new experimental results. Einstein did have something that Newton did not, but it was a theory, not an observation: special relativity. Einstein's main objective was to achieve *theoretical consistency* between gravity and the rest of known physics. It is perhaps all the more amazing, therefore, that in the end Einstein devised a theory that made many new and completely unexpected predictions that could be tested by experiment and astronomical observation.

Our purpose for the rest of this book is to learn about general relativity and its applications. This will take us on a journey to some of the most interesting phenomena in astronomy. We will have to steer a careful course between the rocky shoals of too much mathematical complexity and the becalmed waters of over-simplification. There is a huge amount that can be understood well with the level of mathematics we use in this book, and readers will find that the phenomenology of relativistic gravity can be understood, not just learned about, from the few basic principles that we develop, carefully, in this and the next two chapters.

Here are some of the things we will learn how to do.

- ▷Chapter 18 We shall learn how to reproduce the effects of a Newtonian gravitational field by using Einstein's geometric ideas.
- ▷Chapter 18 We shall see how to work out the gravitational deflection of light, getting the correct relativistic value instead of the Newtonian one we found in Chapter 4.
- ▷Chapter 21 We shall compute the orbit of a planet around a black hole, and show that the orbit is not a closed ellipse but rather a precessing ellipse, describing a rosette pattern over time.
- ▷Chapter 19 We shall learn that the main differences between the predictions of general relativity and Newtonian gravity can be traced to a difference in the *source* of gravity, and in particular the way that pressure helps to create Einstein's gravity.
- ▷Chapter 19 We shall deduce that rotating stars and black holes must produce gravitational accelerations that resemble the magnetic forces of electromagnetism, in that they depend on the *velocity* of the object being accelerated.
- ▷Chapter 20 We shall compute the structure of a neutron star and see why stars that are too heavy must collapse to black holes.
- ▷Chapter 22 We shall compute the effect of a gravitational wave on a detector, and so understand why the new gravitational wave astronomy is so interesting.
- ▷Chapter 23 We shall see how gravity creates some of the most beautiful pictures in astronomy, multiplying and distorting images of distant galaxies and quasars as they pass through gravitational lenses.
- ▷Chapter 25 We shall calculate the history of our expanding Universe back to the Big Bang, learn how the elements hydrogen and helium were made, and speculate on how the huge amount of dark matter in the Universe helped stars and galaxies to form.
- ▷Chapter 27 We shall understand, from the way pressure creates Einstein's gravity, why cosmologists believe that the Universe underwent a period of very rapid expansion at the beginning, and why its expansion may even today be accelerating rather than slowing down.
- ▷Chapter 27 We shall glimpse the links between gravitation theory and the theories of the other fundamental forces in physics, as some of the brightest theorists working in physics today struggle to produce a theory of physics containing all the forces in one unified whole.

This is a tantalizing menu for the remainder of our exploration of gravity, but it also an indication of the broad sweep of applications of general relativity in astronomy today. Einstein's invention, devised purely for mathematical consistency, has become essential for the interpretation of the world we see around us. Gravity, the same everyday gravity that Galileo probed with his inclined planes, is the key to understanding the modern Universe.

These predictions of general relativity are radical enough, but what is even more revolutionary about the theory is the *way* it describes gravity.

> Until Einstein, gravity was thought of as simply a force, like the electric force. Einstein described gravity instead as geometry.

Rather than being a force exerted by one body directly on another, gravity was more indirect: one body would cause space and time to curve, and the other body would move in response to this **curvature**. This is unfamiliar language for us: we are used to the idea of a force, but what does it mean that gravity is geometry? The purpose of this chapter and the next is to help us to understand Einstein's way of thinking about gravity.

...geometry

In this section: we learn what geometry is and why it can be used to explain gravity. The key is a distance measure in spacetime called the interval.

Since Einstein describes gravity in terms of geometry, our natural first question is, what do we mean by the word *geometry*? Consider ordinary spaces we are familiar with, such as the surfaces of spheres, or the **Euclidean plane** as represented by a flat piece of paper. All such spaces are smooth and continuous, but when we speak of their geometry we mean something more: we mean their shape, the distances between points in the space, and so on. We calculate distances typically by using coordinates. For example, if I give you the latitude and longitude of both New York and London, you could in principle calculate the distance between them along a great circle on the Earth's surface. This sort of calculation is routine for airlines.

Now, the latitude and longitude of a city are coordinates that locate it on the Earth, just as the x- and y-coordinates locate points on a graph. We generally need coordinates in order to specify which points (cities) we are talking about, and then we use them to compute the distance. But we know that the distance is something that does not depend on the coordinate system we use. For example, we might use longitude measured, not from the Greenwich meridian, but from (say) a line passing through Disneyland California: we could call this the Mickey Mouse coordinate system for the Earth. Although this would change the values of the longitude coordinate we use to describe every city, it would not change the distances between cities.

We want to describe the geometry of relativity. We have already seen that time and space must both be involved, since both are distorted and even mixed by the Lorentz–Fitzgerald transformation. We must therefore explore the geometry of spacetime, the four-dimensional continuum with three spatial dimensions and one time dimension that is the arena for special and general relativity. The unification of space and time into spacetime is one of the most important conceptual advances that special relativity led physicists to. We define and explore it in the next section.

The geometry of a space, like the Earth's surface, is described by the distances between places, not the coordinates of the places. It is something that is a property only of the space itself. When we study the geometry of special relativity and then of spacetimes with gravity, we will of course have to use coordinates (such as t, x, y, and z) to describe events in the spacetime. But we have seen that in special relativity two different observers will use different coordinates. The geometry of the spacetime must not depend on which observer describes it. So we must find ways of describing the geometry using invariant distances between events.

This invariant will be called the **spacetime-interval**. This is a word we have used often in this book to represent a particular lapse of time. In relativity it is used in a very specific manner, to represent a measure of separation of events in time or space that is agreed by all experimenters, independent of the coordinates of the events. We will define it later in this chapter and then use it repeatedly through the rest of the book. The geometry of spacetime is determined by the spacetime-intervals between events. Spacetimes that describe gravitational fields